SOLVED PAPER 2009

QUESTION NO. 1: SOLUTION

Weekly Wages	No.of Works f	x	fx	fx^2	C.B.	C.F
30-39	6	34.5	207	7141.5	29.5-39.5	6
40-49	10	44.5	445	19802.5	39.5-49.5	16
50-59	11	54.5	599.5	32672.75	49.5-59.5	27
60-69	12 f_m	64.5	774	49923	59.5-69.5	39
70-79	32 f_m	74.5	2384	177608	69.5-79.5	71
80-89	18 f_2	84.5	1521	128524.5	79.5-89.5	89
90-99	8	94.5	756	71442	89.5-99.5	97
Total	$\sum f = 97$		6696.5	487114.25		

(a) Mode

$$= l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \; x \; h$$

$$= 69.5 + \frac{32 - 12}{(32-12) + (32-18)} \; x \; 10$$

$$= 69.5 + \frac{20 \; x \; 10}{20+14}$$

$$= 75.38$$

(b) Median

$$= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 69.5 + \frac{10}{32}(48.5 - 39)$$

$$= 72.47$$

$\frac{n}{2} = \frac{97}{2} = 48.5$

$l = 69.5$, h = 10, C = 39

(c) C.V

$$= \frac{S}{\bar{x}} \text{ x } 100\%$$

where x $= \frac{\sum fx}{\sum f} = \frac{6686.5}{97} = 68.93$

$$S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{487114.25}{97} - \left(\frac{6686.5}{97}\right)^2}$$

$= \sqrt{5021.796 - (68.93)^2}$

$= 16.45$

C.V $= \frac{S}{\bar{x}}$ x 100%

$= \frac{16.45}{68.93}$ x 100%

$= 23.86\%$

QUESTION NO. 2

(a) Consider the year 1950 as base year for the price relatives of commodities A, B, C and D.

Year	Price Relatives				Link Relatives				G.M.	Chain Indices
	A	B	C	D	A	B	C	D		
1950	100	100	100	100	-	-	-	-	-	100
1951	81	77	119	55	81	77	119	55	79.93	79.93
1952	62	54	128	52	76.5	70.1	107.6	94.5	85.93	68.68
1953	104	87	111	100	167.7	161.1	86.7	192.3	145.68	100.06
1954	93	75	154	96	89.4	86.2	138.7	96.0	100.65	100.71
1955	60	43	165	88	64.5	57.3	107.1	91.7	77.62	78.12

(b) Since one card is drawn from 52 playing cards:

n(S) = $\binom{52}{1}$ = 52

(i) A = Black Card

n(A) = $\binom{26}{1}\binom{26}{0}$ $= \frac{1}{2}$

P(A) = $\frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$

(ii) B = Black Card

n(B) = $\binom{16}{1}\binom{36}{0}$ = 16

$$P(B) = \frac{n(B)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

QUESTION NO. 3

X = given population = 2, 4, 6, 10

N = population size = 4

n = Sample size = 2

All possible samples (w.o.r) = $\binom{N}{n} = \binom{4}{2} = 6$

Sr.No.	Samples	Means ($\bar{x}$)
1	2, 4	3
2	2, 6	4
3	2, 10	6
4	4, 6	5
5	4, 10	7
8	6, 10	8

Probability distribution of samples means:

$\bar{x}$	Tally Sheet	f	$P(\bar{x}) = \frac{f}{\sum f}$	$\bar{x}\,p(\bar{x})$	$\bar{x}^2\,p(\bar{x})$
3	1	1	1/6	3/6	9/6
4	1	1	1/6	4/6	16/6
5	1	1	1/6	5/6	25/6
6	1	1	1/6	6/6	36/6
7	1	1	1/6	7/6	49/6
8	1	1	1/6	8/6	64/6
Total		$\sum f = 6$		33/6	199/6

Mean and variance of sampling distribution of means:

$$\mu_{\bar{x}} = \sum \bar{x}P(\bar{x}) = \frac{33}{6} = 5.5$$

$$\sigma_{\bar{x}}^2 = \sum \bar{x}^2 P(\bar{x}) - (\mu_{\bar{x}})^2$$

$= \frac{199}{6} - (5.5)^2$

$= 2.92$

Mean and Variance of population:

x	x^2
2	4
4	16
6	36
10	100
$\sum x = 22$	$\sum x^2 = 156$

$\mu = \frac{\sum x}{N} = \frac{22}{4} = 5.5$

$\sigma^2 = \frac{\sum x^2}{n} - \mu^2$

$= \frac{156}{4} - (5.5)^2$

$= 8.76$

Verification:

(i) $\mu_{\bar{x}} = \mu$

$5.5 = 5.5$

(ii) $\sigma_{\bar{x}}^2 = \frac{\sigma 2}{n} \cdot \frac{N-n}{N-1}$

$2.92 = \frac{8.75}{2} x \frac{4-2}{4-1}$

$2.92 = 2.92$

QUESTION NO. 4

(a) H_0: There is no association between general ability and mathematical ability.

H_1: There is some association between general ability and mathematical ability.

Level of significance: $\alpha = 0.05$, $1 - \alpha = 0.95$

Test statistics: $x^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

Degrees of freedom: v = (r – 1) (c – 1)

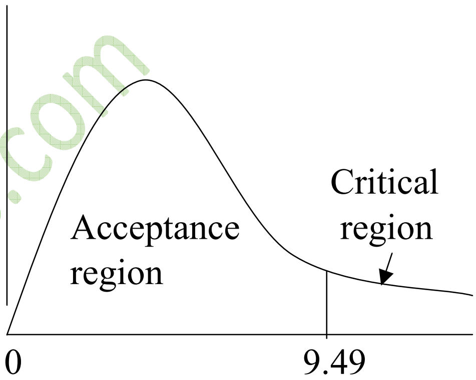
=(3 – 1) (3 – 1)

= 2 x 2 = 4

Critical Value: $x^2_{n,1-\alpha} = x^2_{4,0.95} = 9.49$

Critical region: $x^2 > 9.49$

Decision rule: Reject H_0, if $x^2 > 9.49$,
Otherwise accept H_0.



Observed Frequency (O_{ij})

Mathematical Ability	General Ability			Total
	Good	Fair	Poor	
Good	44	22	4	70
Fair	265	257	178	700
Poor	41	91	98	230
Total	350	370	280	1000

o_{ij}	e_{ij}	$o_{ij} - e_{ij}$	$(o_{ij} - e_{ij})^2$	$(o_{ij} - e_{ij})^2 / e_{ij}$
44	24.5	19.5	380.25	15.5204
265	245	20	400	1.6327
41	80.5	-39.5	1560.25	19.3820
22	25.9	-3.9	15.21	0.5873
257	259	-2	4	0.0154
91	85.1	5.9	34.81	0.4090
4	19.6	-15.6	243.36	12.4163

178 98	196 64.4	-18 33.6	324 1128.96	1.6531 17.5304
Total				69.1466

Conclusion: Since $x^2 = 69.1466 > x^2_{4,0.95} = 9.49$

So, we reject H_0.

Regression coefficient of y on x:

$$b_{yx} = \frac{n\sum xy - \sum x.\sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{8\times 94.7 - 17.6\times 32.8}{8\times 49.64 - (17.6)^2}$$

$$= \frac{180.32}{87.36} = 2.064$$

Regression coefficient of x on y:

$$b_{xy} = \frac{n\sum xy - \sum x.\sum y}{n\sum y^2 - (\sum y)^2}$$

$$= \frac{8\times 94.7 - 17.6\times 32.8}{8\times 182 - (32.8)^2}$$

$$= \frac{160.32}{380.16} = 0.474$$

QUESTION NO. 5

a) Solve the Equation for x

$\sqrt{5x-4} - \sqrt{3x+1} = 1$

Taking square on both sides:

$(\sqrt{5x-4} - \sqrt{3x+1})^2 = (1)^2$

$(5x + 4) + (3x + 1) - 2\sqrt{5x-4}\sqrt{3x+1} = 1$

$5x + 4 + 3x + 1 - 2\sqrt{(5x-4)(3x+1)} = 1$

$8x + 5 - 2\sqrt{15x^2 + 5x + 12x + 4} = 1$

$-2\sqrt{15x^2 + 17x + 4} = 1 - 8x - 5$

$-2\sqrt{15x^2 + 17x + 4} = -4 - 8x$

$-2\sqrt{15x^2 + 17x + 4} = -2(2 + 4x)$

$\sqrt{15x^2 + 17x + 4} = 2 + 4x$

Again, taking square on both sides:

$15x^2 + 17x + 4 = (2 + 4x)^2$

$15x^2 + 17x + 4 = 4 + 16x^2 + 16x$

$-x^2 + x = 0$

$-x(x - 1) = 0$

Solution set is {0, 1}

(b) $\frac{x+1}{3x} = \frac{1}{x} - \frac{1}{3}$

$\frac{x+1}{3x} = \frac{3-1}{3x}$

$x + 1 = 3 - x$

$x + x = 3 - 1$

$2x = 2$

$x = 1$

Solution set is {1}

QUESTION NO. 6

(a)

$2x + 6y + 4z$	=	320	…..(i)
$6x + 6y + 4z$	=	480	…..(ii)
$3x + 2y + 4z$	=	192	…..(iii)

Subtract equation (i) from (ii), we get:

$6x + 6y + 4z = 480$

$-2x \pm 6y \pm 4z = -320$

$4x = 160$

$x = 160/4 = 40$

Subtract equation (iii) from (ii), we get:

$6x + 6y + 4z = 480$

$-3x \pm 2y \pm 4z = 192$

$3x + 4y = 288$ …..(iv)

Put x = 40 in eq. (iv):

$$3(40) + 4y = 288$$
$$4y = 288 - 120$$
$$y = 42$$

Put x = 40 and y = 42 in eq. (i):

$$2(40) + 6(42) + 4z = 320$$
$$4z = 320 - 120$$
$$z = -3$$

Solution set is {(40, 42, – 3)}

(b) We have: $a_{10} = 20$ and $a_{20} = 40$, Find a_7 of the A.P.

Since $a_n = a + (n - 1)d$

$$a_{10} = a + (10 - 1)d$$
$$20 = a + 9d \quad .....(i)$$
$$a_{20} = a + (20 - 1)d$$
$$40 = a + 19d \quad .....(ii)$$

Subtract equation (i) from (ii), we get:

$$40 = a + 19d$$
$$\underline{-20 = -a \pm 9d}$$
$$20 = 10d$$
$$d = 2$$

Put d = 2 in eq. (i)

$20 = a + 9(2)$

$a = 20 - 18 = 2$

$a = 2$

Now, 7th term of the A.P. is

$a_7 = a + (7 - 1)d$

$= 2 + 6 \times 2 = 14$

QUESTION NO. 7

(a) P = Principal amount = ?

i = Internal rate = 5% p.a. = 0.05

n = No. of periods = 3 years

Now, difference between compound interest and simple interest = Rs. 61

$P[(1 + i)^n - 1] - P \times i \times n = \text{Rs. } 61$

$P[(1 + 0.05)^3 - 1] - P \times 0.05 \times 3 = 61$

$P[(1.05)^3 - 1] - 0.15P = 61$

$0.157625P - 0.15P = 61$

$0.007625P = 61$

$P = \frac{61}{0.007625}$

$P = \text{Rs. } 8000$

Thus, principal amount is Rs. 8000.

(b) R = Rs. 5000 (Payable at the end of the each quarter. It is ordinary annuity) n = 5 years = 5 x 4 = 20 quarters

i = 8% p.a. = $\frac{0.08}{4}$ = 0.02 per quarters

The accumulated value is:

$$S_n = R\frac{(1-i)^n-1}{i}$$

$$= 5000\frac{(1-0.02)^{20}-1}{0.02}$$

$$= 5000(24.29737) = \text{Rs. } 121486.85$$

QUESTION NO. 8

(i) A matrix is defined as the set of real numbers arranged in the form of rectangular array of numbers enclosed in brackets. Generally, matrices are represented by capital letters such as A, B. c. e etc. For example:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 46 \\ 8 & 01 \end{bmatrix} \text{ etc.}$$

(ii) A specific number which is multiplied to every next term in a geometric sequence. It is represented by "r".

(iii) Compound interest is an interest paid on the initial principal and previously earned interest.

$$C.I = P[(1+i)^n - 1]$$

where C.I = Compound interest

i = Interest rate

n = Number of periods

(iv) When the payments are made at beginning of each period and continue for a definite period, it is called annuity due.

Sum of annuity due:

$$S_n = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$

Where R = Regular installment

i = Interest rate

n = Number of periods

(v) The totality of observation in particular situation is called population.

(vi) A sample is a subgroup of the population that will represent the characteristics of the population whereas sampling is the procedure of selecting a representative sample from a given population.

(vii) Correlation is a measure of the degree to which any two variables vary together.

(viii) The square root of the average of all squared deviations taken from A.m. is called standard deviation.

$$S = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad \text{and} \quad S = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

(ix) The tendency of the values to concentrate at their centre is called central tendency and any measure indicating the centre of their distribution is called measured central tendency.

(x) If $x_1, x_2, \ldots\ldots, x_n$ are n observations with their respective weights $w_1, w_2, \ldots\ldots, w_n$. Then weighted mean is defined as:

$$\overline{x_n} = \frac{x_1w_1+x_2w_2+\ldots\ldots+x_nw_n}{w_1+w_2+\ldots\ldots+w_n}$$

$$= \frac{\sum x_w}{\sum w}$$