

SOLVED PAPER 2011

QUESTION NO. 1: SOLUTION

Classes	f	x	D	fD	fD ²	C.f
12.5-17.5	2	15	-20	-40	800	3
17.5-22.5	22	20	-15	-230	4750	24
22.5-27.5	19	25	-10	-190	1900	43
27.5-32.5	14	30	-5	-70	350	57
32.5-37.5	8	35	-5	-70	350	57
37.5-42.5	4	40	5	20	100	64
42.5-47.5	6	45	10	60	600	70
47.5-52.5	1	50	15	15	225	71
52.5-57.5	1	55	20	20	400	72
	75			-525	9325	

$$P.M = 35 \quad , \quad \frac{n}{2} = \frac{72}{2} = 36$$

$$\begin{aligned} \bar{x} &= P.M. + \frac{\sum fD}{\sum f} \\ &= 35 + \frac{-515}{72} \\ &= 35 - 7.153 \\ &= 27.847 \end{aligned}$$

$$\begin{aligned} \text{Median} &= l + \frac{h}{f} \left(\frac{n}{2} - c \right) \\ &= 22.5 + \frac{5}{19} (36 - 24) \\ &= 22.5 + \frac{5}{19} (12) \\ &= 22.5 + \frac{60}{19} \\ &= 25.658 \end{aligned}$$

$$\begin{aligned}
 \text{S.D} &= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \\
 &= \sqrt{\frac{9325}{72} - \left(\frac{-515}{72}\right)^2} \\
 &= \sqrt{129.5139 - 51.1622} \\
 &= \sqrt{78.3517} \\
 &= 8.851649564 \quad \text{or} \quad 8.852
 \end{aligned}$$

$$\begin{aligned}
 \text{C.V} &= \frac{\text{S.D}}{\text{A.M}} \times 100\% \\
 &= \frac{8.852}{27.847} \times 100\% \\
 &= 31.788\% \quad \text{or} \quad 31.79\%
 \end{aligned}$$

QUESTION NO. 2.

x	y	x ²	y ²	xy
5	9	25	81	45
6	7	36	49	42
7	10	49	100	70
8	3	64	9	24
9	13	81	169	117
10	11	100	121	110
11	14	121	196	154
12	10	144	100	120
13	14	169	196	182
14	12	196	144	168
15	18	225	324	270
110	121	1210	1489	1308

$$\bar{x} = \frac{\sum x}{n} = \frac{110}{11} = 10$$

$$\bar{y} = \frac{\sum y}{n} = \frac{121}{11} = 11$$

$$\begin{aligned}\sigma_x &= \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \\ &= \sqrt{\frac{1210}{11} - \left(\frac{110}{11}\right)^2} \\ &= \sqrt{110 - 100} \\ &= 3.162\end{aligned}$$

$$\begin{aligned}\sigma_y &= \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2} \\ &= \sqrt{\frac{1489}{11} - \left(\frac{121}{11}\right)^2} \\ &= \sqrt{135.3636 - 121} \\ &= \sqrt{14.3636} \\ &= 3.79\end{aligned}$$

$$\begin{aligned}r &= \frac{\sum xy - n.\bar{x}.\bar{y}}{n.\sigma_x.\sigma_y} \\ &= \frac{130.2 - 11 \times 10 \times 11}{11 \times 3.162 \times 3.79} \\ &= \frac{92}{131.824} \\ &= 0.698\end{aligned}$$

Line of regression y on x:

$$\begin{aligned}y - \bar{y} &= r \frac{\sigma_x}{\sigma_y} (x - \bar{x}) \\ y - 11 &= 0.698 \times \frac{3.79}{3.162} (x - 10) \\ &= 0.837x - 8.37 \\ y &= 0.837x - 8.37 + 11 \\ y &= 0.837x + 2.63\end{aligned}$$

QUESTION NO. 3

Given population = 3, 6, 9, 12, 15, 18

$$N = \text{population size} = 6$$

$$n = \text{Sample size} = 3$$

All possible samples without replacement = ${}^6C_3 = 20$

$$\frac{6!}{(6-3)! \cdot 3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1!} = 20$$

$$x: \quad 3 + 6 + 9 + 12 + 15 + 18 = 63$$

$$x^2: \quad 9 + 36 + 81 + 144 + 225 + 324 = 819$$

$$\text{Population mean} = \frac{\sum x}{N} = \frac{63}{6} = 10.5$$

$$\begin{aligned} \text{Population standard deviation} &= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{n}\right)^2} \\ &= \sqrt{\frac{819}{6} - \left(\frac{63}{6}\right)^2} \\ &= \sqrt{136.5 - 110.25} \\ &= \sqrt{26.25} \\ &= 5.123 \end{aligned}$$

Sample: (3,6, 9), (3, 6, 12), (3, 6, 15), (3, 6,18), (3, 9, 12), (3, 9, 15), (3, 9, 18)

Sample mean: 6, 7, 8, 9, 8, 9, 10

Sample: (3, 12, 15), (3, 12, 18), (3, 15, 18), (6,9, 12), (6,9, 15), (6,9, 18), (6,12,15)

Sample mean: 10, 11, 12, 9, 10, 11, 11,

Sample: (6, 12, 18), (6, 15, 18), (9, 12, 15), (9, 12, 18), (9, 15, 18), (12, 15, 18)

Sample mean: 12, 13, 12, 13, 14, 15

Sample \bar{x}	Tally	f	$P(\bar{x}) = \frac{f}{\Sigma f}$	$\bar{x} p(\bar{x})$	$\bar{x}^2 p(\bar{x})$
6	1	1	1/20	6/20	36/20
7	1	1	1/20	7/20	49/20
8	11	2	2/20	16/20	128/20
9	111	3	3/20	27/20	243/20
10	111	3	3/20	30/20	300/20
11	111	3	3/20	33/20	363/20
12	111	3	3/20	36/20	432/20
13	11	2	2/20	26/20	338/20
14	1	1	1/20	4/20	196/20
15	1	1	1/20	15/20	225/20
Total		20	1	210/20	2310/20

Mean of sampling distribution of means:

$$\Sigma \bar{x} P(\bar{x}) = \frac{210}{20} = 10.5$$

$$\sigma = \sqrt{\Sigma \bar{x}^2 P(\bar{x}) - [\Sigma \bar{x} P(\bar{x})]^2}$$

$$= \sqrt{\frac{2310}{20} - \left(\frac{210}{20}\right)^2}$$

$$= \sqrt{115.5 - (10.5)^2}$$

$$= \sqrt{115.5 - 110.25}$$

$$= \sqrt{5.25} = 2.291$$

QUESTION NO. 4

Commodity	Prices		Quantity		p_0q_0	p_0q_1	p_1q_0	p_1q_1
	1995	2002	1995	2002				
	p_0	p_1	q_0	q_1				
A	60	80	270	290	16200	17400	21600	23200
B	40	45	125	140	5000	5600	5625	6300
C	20	25	130	140	2600	2800	3250	3500
D	55	70	270	350	14850	19250	18900	24500
Total					38650	45050	49375	57500

Price index number by:

(i) Laspeyr's
$$= \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

$$= \frac{49375}{38650} \times 100$$

$$= 127.75$$

(ii) Fisher's
$$= \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$$

$$= \sqrt{\frac{49375}{38650} \times \frac{57500}{45050}} \times 100$$

$$= \sqrt{1.630537006} \times 100$$

$$= 127.69$$

(iii) Paasche's
$$= \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

$$= \frac{57500}{45050} \times 100$$

$$= 127.64$$

(iv) Marahall
$$= \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

$$\begin{aligned} &= \frac{49375+57500}{38650+45050} \times 100 \\ &= \frac{106875}{83700} \times 100 \\ &= 127.69 \end{aligned}$$

QUESTION NO. 5

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 0 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 5 & 4 \\ 0 & 3 & -5 \\ -1 & 4 & 2 \end{bmatrix}$$

$$(i) \quad A - 3B = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 0 \\ 4 & 5 & 6 \end{bmatrix} - 3 \begin{bmatrix} -2 & 5 & 4 \\ 0 & 3 & -5 \\ -1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 3-15 & 2-12 \\ 3-0 & 2-9 & 0+15 \\ 4+3 & 5-13 & 6-6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 & -10 \\ 3 & -7 & 15 \\ 7 & -7 & 0 \end{bmatrix}$$

$$(i) \quad AB = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 2 & 0 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} -2 & 5 & 4 \\ 0 & 3 & -5 \\ -1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0-2 & 5+9+8 & 4-15+4 \\ -6+0+0 & 15+6+0 & 12-10+0 \\ -8+0-6 & 20+15+24 & 16-25+12 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 22 & -7 \\ -6 & 21 & 2 \\ -14 & 59 & 3 \end{bmatrix}$$

QUESTION NO. 6

(a) $x^2 + 5x = 50$
 $x^2 + 10x - 5x - 50 = 0$
 $x(x + 10) - 5(x + 10) = 0$
 $x + 10 = 0$ or $x - 5 = 0$
 $x = -10$ or $x = 5$

or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $= \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times (-50)}}{2 \times 1}$
 $= \frac{-5 \pm 15}{2}$
 $= \frac{-5 - 15}{2} = \frac{-20}{2} = -10$
 $= \frac{-5 + 15}{2} = \frac{10}{2} = 5$

(OR)

$$x^2 + 5x = 50$$

$$x^2 + 2\left(\frac{5}{2}x\right) = 50$$

$$x^2 + 2\left(\frac{5}{2}x\right) + \left(\frac{5}{2}\right)^2 = 50 + \left(\frac{5}{2}\right)^2 = 56.25$$

$$\left(x + \frac{5}{2}\right)^2 = 56.25 = (\sqrt{56.25})^2$$

$$x + \frac{5}{2} = \sqrt{56.25} = \pm 7.5$$

$$x = 7.5 - \frac{5}{2} = 5$$

$$x = -7.5 - \frac{5}{2} = -10$$

(b) Suppose 1st even integer = x

Then 2nd even integer = x + 2

Sum of the two even interger = x + (x + 2) = 66

$$x + (x + 2) = 66$$

$$2x = 66 - 2$$

$$2x = 64$$

$$x = \frac{64}{2} = 32$$

So the 1st integer = 32

2nd integer = x + 2

$$= 32 + 2$$

$$= 34$$

(32, 34)

QUESTION NO. 7

(a) Given for A.P.:

$$a_{54} = -61 \quad \text{and} \quad a_4 = 64$$

$$a_x = a + (x - 1)d$$

$$a_{54} = a + (54 - 1)d = -61$$

$$a + 53d = -61 \quad \dots\dots(i)$$

$$a_4 = a + (4 - 1)d = 64$$

$$\begin{array}{rcl}
 a + 3d & = & 64 & \dots\dots(ii) \\
 a + 53d & = & -61 & \dots\dots(i) \\
 \hline
 -50d & = & 125 \\
 d & = & \frac{125}{-50} = -2.5
 \end{array}$$

By putting the value of d in equation (i), we get:

$$\begin{aligned}
 a + 53d &= -61 \\
 a + 53(-2.5) &= -61 \\
 a + (-132.5) &= -61 \\
 a &= -61 + 132.5 = 71.5 \\
 a_{23} &= a + (23 - 1)d \\
 &= 71.5 + 22(-2.5) \\
 &= 71.5 - 55 \\
 &= 16.5
 \end{aligned}$$

(b) Sum of $0.53 + 0.0053 + 0.000053 + \dots$ to infinity

Where $a = 0.53$, $r = \frac{0.0053}{0.53} = 0.01 < 1$

So,

$$\begin{aligned}
 S_n &= \frac{a}{1-r} \\
 &= \frac{0.53}{1-0.01} \\
 &= \frac{0.53}{0.99} \\
 &= \frac{53}{99}
 \end{aligned}$$

QUESTION NO. 8

(a) Suppose the original value of property = x

$$1\text{st year loss} = x \left(\frac{10}{100}\right) = 0.10x$$

$$\text{Remaining value after 1st loss} = x - 0.10x = 0.90x$$

$$2^{\text{nd}} \text{ year loss} = 0.90x \times 10\% = 0.09x$$

$$\text{Remaining value 2nd loss} = 0.90x - 0.09x = 0.81x$$

$$3^{\text{rd}} \text{ year loss} = 0.81x \times 10\% = 0.081x$$

$$0.081x = 202.5 \quad \text{or} \quad x = \frac{202.5}{0.081} = \text{Rs. } 2500$$

(b) Given: Compound interest – Simple interest = Rs. 31

$$3 \text{ years simple interest} = x \left(\frac{10}{100}\right)(3) = 0.300x$$

$$\begin{aligned} 3 \text{ year compound interest} &= x \left(1 + \frac{10}{100}\right)^3 - x = x(1.10)^3 - x \\ &= 1.331x - x = 0.331x \end{aligned}$$

$$0.331x - 0.300x = \text{Rs. } 31$$

$$0.031x = \text{Rs. } 31$$

$$x = \frac{31}{0.031} = \text{Rs. } 1000$$