

Solved Paper 2012**QUESTION NO. 1: SOLUTION**

x	f	f.x	D'	fD'	fD ²	Groups	C.f
115	6	690	-4	-24	96	110-120	6
125	25	3125	-3	-75	225	120-130	31
135	48	6480	-2	-96	192	130-140	79
145	72	10440	-1	-72	72	140-150	151
155	116	7980	0	0	0	150-160	267
165	60	9900	1	60	60	160-170	327
175	38	6650	2	76	152	170-180	365
185	22	4070	3	66	18	180-190	387
195	2	390	4	8	32	190-200	389
-	389	59725	-	-57	1027	-	-

$$(i) \quad \bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{59725}{389} = 153.535$$

$$\bar{x} = P.M. + \frac{\sum fD'}{\sum f} \times i$$

$$= 155 + \frac{-57}{389} \times 10$$

$$= 155 - 1.465 = 153.535$$

$$(ii) \quad \frac{n}{2} = \frac{389}{2} = 194.5$$

$$\frac{n}{4} = \frac{389}{4} = 97.25$$

$$\frac{3n}{4} = \frac{3 \times 389}{4} = 291.75$$

$$\text{Mode} = l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times i$$

$$= 150 + \frac{116 - 7}{(116 - 7) + (116 - 60)} \times 10$$

$$= 150 + \frac{440}{100}$$

$$= 150 + 4.4 = 154.4$$

$$(iii) \quad \text{Median} = l + \frac{i}{f} \left(\frac{n}{2} - c \right)$$

$$= 150 + \frac{10}{116} (194.5 - 151)$$

$$= 150 + 3.75 = 153.75$$

$$Q_1 = l + \frac{i}{f} \left(\frac{n}{4} - c \right)$$

$$= 140 + \frac{10}{72} (94.25 - 79)$$

$$= 140 + \frac{10 \times 18.25}{72}$$

$$= 140 + 2.535 = 142.535$$

$$Q_3 = l + \frac{i}{f} \left(\frac{3n}{4} - c \right)$$

$$= 160 + \frac{10}{60} (291.75 - 267)$$

$$= 160 + \frac{10 \times 24.75}{60}$$

$$= 160 + 0.4125 = 160.4125$$

$$\sigma = \sqrt{\frac{\sum fD'^2}{\sum f} - \left(\frac{\sum fD'}{\sum f} \right)^2} \times i$$

$$= \sqrt{\frac{1027}{389} - \left(\frac{-57}{389} \right)^2} \times 10$$

$$= \sqrt{2.6401 - 0.02154} \times 10$$

$$= \sqrt{2.6186} \times 10$$

$$= 1.6182 \times 10 = 16.182$$

$$\text{Or} \quad \text{S.K} = \frac{3(\text{Mean} - \text{Median})}{S.D.}$$

$$= \frac{3(153.535 - 153.75)}{16.182}$$

$$\begin{aligned} &= \frac{3(-0.215)}{16.182} \\ &= \frac{-0.645}{16.182} = -0.039859102 \text{ or } -0.04 \end{aligned}$$

Or S.K

$$\begin{aligned} &= \frac{Q_3 + Q_1 - 2\text{Median}}{Q_3 - Q_1} \\ &= \frac{160.4125 + 142.535 - (2 \times 153.75)}{160.4125 - 142.535} \\ &= \frac{302.9475 - 307.5}{17.8875} \\ &= \frac{-4.5525}{17.8875} = -0.2546 \text{ or } -0.255 \end{aligned}$$

QUESTION NO.2: SOLUTION

(a) $r = \frac{\frac{\sum xy}{n} - \frac{\sum x}{n} \times \frac{\sum y}{n}}{\sigma_x \sigma_y}$

$$\begin{aligned} &= \frac{\frac{132}{4} - \frac{20}{4} \times \frac{25}{4}}{\sqrt{2.5} \times \sqrt{1.4375}} \\ &= \frac{33 - 31.25}{\sqrt{3.59375}} \\ &= \frac{1.75}{1.8957} = 0.923 \end{aligned}$$

$$\begin{aligned} \bar{x} &= \frac{\sum x}{n} & \bar{y} &= \frac{\sum y}{n} \\ &= \frac{20}{4} = 5 & &= \frac{25}{4} = 6.25 \end{aligned}$$

$$\begin{aligned} \sigma_x &= \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} \\ &= \sqrt{\frac{110}{4} - \left(\frac{20}{4}\right)^2} \\ &= \sqrt{27.5 - 25} \\ &= \sqrt{2.5} = 1.581 \end{aligned}$$

$$\begin{aligned}
 \sigma_x &= \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2} \\
 &= \sqrt{\frac{162}{4} - \left(\frac{25}{4}\right)^2} \\
 &= \sqrt{40.5 - 39.0625} \\
 &= \sqrt{1.4375} \\
 &= 1.19896 \text{ or } 1.199
 \end{aligned}$$

Regression Equation y on x:

$$\begin{aligned}
 y - \bar{y} &= r \frac{\sigma_x}{\sigma_y} (x - \bar{x}) \\
 y - 6.25 &= 0.923 \times \frac{1.199}{1.581} (x - 5) \\
 &= 0.7x - 3.5 \\
 y &= 0.7x - 3.5 + 6.25 \\
 &= 0.7x + 2.75
 \end{aligned}$$

(b)

Commodity	Base year		Current year		p_1q_0	p_0q_0	p_1q_1	p_0q_1
	p_0	q_0	p_1	q_1				
A	6	140	8	150	1120	840	1200	900
B	10	160	12	180	1920	1600	2160	1800
C	16	80	20	110	1600	1280	2200	1760
D	20	100	24	120	2400	2000	2880	2400
Total					7040	5720	8440	6860

$$\begin{aligned}
 \text{Laspeyr's index} &= \frac{\sum p_1q_0}{\sum p_0q_0} \times 100 \\
 &= \frac{7040}{5720} \times 100 = 123.08
 \end{aligned}$$

$$\begin{aligned}
 \text{Paasche's index} &= \frac{\sum p_1q_1}{\sum p_0q_1} \times 100 \\
 &= \frac{8440}{6860} \times 100 = 123.03
 \end{aligned}$$

$$\begin{aligned} \text{Fisher's index} &= \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100 \\ &= \sqrt{\frac{7040}{5720} \times \frac{8440}{6860}} \times 100 \\ &= \sqrt{1.514240861} \times 100 = 123.05 \end{aligned}$$

$$\begin{aligned} \text{MarahallEdgeworth index} &= \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100 \\ &= \frac{7040 + 8440}{5720 + 6860} \times 100 \\ &= \frac{15480}{12580} \times 100 = 123.05 \end{aligned}$$

QUESTION NO. 3: SOLUTION

Observed Frequency (f_o)					
Attribute	A ₁	A ₂	A ₃	A ₄	Total
B ₁	42	83	72	72	269
B ₂	33	62	82	64	241
B ₃	37	121	93	90	341
Total	112	266	247	226	851

Expected Frequency (f_e)					
Attribute	A ₁	A ₂	A ₃	A ₄	Total
B ₁	35.40	84.08	78.08	71.44	269
B ₂	31.72	75.33	68.95	64.00	241
B ₃	44.88	106.59	98.97	92.56	341
Total	112	266	247	226	851

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2 / f_e$
42	35.40	6.60	43.56	1.23051
33	31.72	1.28	1.6384	0.05165
37	44.88	-7.88	62.0944	1.38356
83	84.08	-1.08	1.1664	0.01387
62	75.33	-13.33	177.6889	2.35887
121	10.59	14.41	207.6481	1.94810
72	78.08	-6.08	36.9664	0.47344

82	69.95	12.05	145.2025	2.07580
93	98.97	-5.97	35.6409	0.36012
72	71.44	0.56	0.3136	0.00439
64	64.00	0	0.0000	0.00000
90	90.56	-0.56	0.3136	0.00346
				9.90377

Chi-square = $\sum \frac{(f_o - f_e)^2}{f_e} = 9.90377$

Calculated value of chi-square 9.90 is less than the table value i.e. 12.59, therefore, there is no association between the two classifications and accept null hypothesis.

(i)

x	d	d ²
2	-4	16
4	-2	4
6	0	0
8	2	4
10	4	16
30		40

Mean of population = $\frac{\sum x}{n} = \frac{30}{5} = 6$

Variance of population = $\frac{\sum d^2}{n} = \frac{40}{6} = 6.667$

(ii) Mean and variance of each sample = $\frac{(x-x)^2}{n-1}$

Samples	Mean	Calculation of variance
2, 4	3	$\frac{(2-3)^2 + (4-3)^2}{2-1} = 2$
2, 6	4	$\frac{(2-4)^2 + (6-4)^2}{2-1} = 8$
2, 8	5	$\frac{(2-5)^2 + (8-5)^2}{2-1} = 18$
2, 10	6	$\frac{(2-6)^2 + (10-6)^2}{2-1} = 32$

4, 6	5	$\frac{(4-5)^2+(6-5)^2}{2-1} = 2$
4, 8	6	$\frac{(4-6)^2+(8-6)^2}{2-1} = 8$
4, 10	7	$\frac{(4-7)^2+(10-7)^2}{2-1} = 18$
6, 8	7	$\frac{(6-7)^2+(8-7)^2}{2-1} = 2$
6, 10	8	$\frac{(6-8)^2+(10-8)^2}{2-1} = 8$
8, 10	9	$\frac{(8-9)^2+(10-9)^2}{2-1} = 2$

(iii) Average of all (mean) samples or mean of all samples:

$$= \frac{3+4+5+6+5+6+7+7+8+9}{10} = \frac{60}{10} = 6$$

Average of the variance of all samples:

$$= \frac{2+8+18+32+2+8+18+2+8+2}{10} = \frac{100}{10} = 10$$

QUESTION NO. 5: SOLUTION

$$\text{Inverse of } A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = A^{-1} = \frac{1}{|A|} \text{adj. (A)}$$

Co-factors of A are:

$$A_{11} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1(2-3) = -1$$

$$A_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = -1(1-9) = 8$$

$$A_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1(1-6) = -5$$

$$A_{21} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -1(1-3) = 2$$

$$A_{22} = (-1)^4 \begin{vmatrix} 0 & 3 \\ 3 & 1 \end{vmatrix} = 1(0-9) = -9$$

$$A_{23} = (-1)^5 \begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} = 1(0-3) = 3$$

$$A_{31} = (-1)^4 \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 1(3-6) = -3$$

$$A_{32} = (-1)^5 \begin{vmatrix} 0 & 3 \\ 1 & 3 \end{vmatrix} = -1(0-3) = 3$$

$$A_{33} = (-1)^6 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(0-1) = -1$$

$$\begin{aligned} |A| &= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} \\ &= 0 \times (-1) + 1 \times 8 + 3 \times (-5) \\ &= 0 + 8 - 15 \\ &= -7 \neq 0 \end{aligned}$$

$$\text{Co-factor } A = \begin{bmatrix} -1 & 8 & -5 \\ 2 & -9 & 3 \\ -3 & 3 & -1 \end{bmatrix} \text{ in } \text{adj } A = \begin{bmatrix} -1 & 2 & -3 \\ 8 & -9 & 3 \\ -5 & 3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$= \frac{-1}{7} \begin{bmatrix} -1 & 2 & -3 \\ 8 & -9 & 3 \\ -5 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{7} & \frac{-2}{7} & \frac{3}{7} \\ \frac{-8}{7} & \frac{9}{7} & \frac{-3}{7} \\ \frac{5}{7} & \frac{-3}{7} & \frac{1}{7} \end{bmatrix}$$

QUESTION NO. 6 : SOLUTION

(a) $x = \sqrt{x+3} - 3$
 $x - 3 = \sqrt{x+3}$
 $(x-3)^2 = (\sqrt{x+3})^2$
 $x^2 + 6x + 9 = x + 3$
 $x^2 + 6x + 9 - x - 3 = 0$
 $x^2 + 5x + 6 = 0$
 $x^2 + 3x + 2x + 6 = 0$
 $x(x+3) + 2(x+3) = 0$
 $(x+2)(x+3) = 0$
 $(x+2) = 0$ or $(x+3) = 0$
 $x = -2$ or $x = -3$

(b) (i) $9x + 15y = 123$

(ii) $15x + 93y = 201$

Or (i) $15x + 25y = 205$

(ii) $\underline{15x + 93y = 201}$

$$-68y = 4$$

$$y = \frac{4}{-68} = \frac{-1}{17}$$

By putting the value of y in equation (i):

$$9x + 15y = 123$$

$$9x + 15x \frac{-1}{17} = 123$$

$$9x - \frac{15}{17} = 123$$

$$9x = 123 + \frac{15}{17}$$

$$9x = \frac{2091+15}{17}$$

$$x = \frac{2091+15}{17 \times 9}$$

$$x = \frac{234}{17}$$

$$x = \frac{234}{17} \quad \text{and} \quad y = \frac{-1}{17}$$

QUESTION NO. 7: SOLUTION

(a)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_6 = \frac{\frac{1}{3} \left(1 - \left(-\frac{1}{3} \right)^6 \right)}{1 - \left(-\frac{1}{3} \right)}$$

$$= \frac{\frac{1}{3} \left(1 - \frac{1}{729} \right)}{1 + \left(\frac{1}{3} \right)}$$

$$= \frac{\frac{1}{3} \times \frac{728}{729}}{\frac{4}{3}}$$

$$= \frac{182}{729}$$

(b)

$$S_n = \frac{n}{2}(a + L)$$

$$400 = \frac{n}{2}(5 + 45)$$

$$400 = \frac{n}{2}(50) = 25n$$

$$n = \frac{400}{25} = 16 \text{ terms}$$

$$L = a + (n - 1)d$$

$$45 = 5 + (16 - 1)d$$

$$45 = 5 + 15d$$

$$15d = 45 - 5 = 40$$

$$d = \frac{40}{15} = \frac{8}{3}$$

QUESTION NO. 8: SOLUTION

Maturity value of A = $(P)(1 + rt)$

$$A = 1500 \left(1 + \frac{6}{100} \times \frac{3}{12} \right)$$

$$= 1500(1.015)$$

$$= \text{Rs. } 15225$$

Discounted value:

$$P = (A)(1 - rt)$$

$$P = 15225 \times \left(1 - \frac{7}{100} \times \frac{75}{360} \right)$$

$$P = 15225 \times \left(1 - \frac{7}{480} \right)$$

$$= 1500 \times \frac{473}{480}$$

$$P = \text{Rs. } 15002.97 \quad \text{or} \quad \text{Rs. } 15003$$