SOLVED PAPER 2013

QUESTION NO. 1

Weight	f	x	fx	fx^2
118-126	20	122	2440	297680
127-135	35	131	4585	600635
136-144	49	140	6860	960400
145-153	32	149	4768	710432
154-162	25	158	3950	624100
163-171	14	167	2338	390446
	175		**$\sum fx = 24941$**	**$\sum fx^2 = 3583693$**

Pearson's coefficient of skewness $= \frac{3(Mean-Median)}{S.D}$

$$= \frac{3(142.52-141.47)}{12.896}$$

$$= \frac{3.15}{12.896}$$

$$= 0.244$$

Bowley's coefficient of skewness $= \frac{Q_3+Q_1-2Median}{Q_3-Q_1}$

$$= \frac{152.16+132.61-2(141.47)}{152.16-132.61}$$

$$= \frac{1.834}{19.554}$$

$$= 0.0938$$

Arithmetic mean $= \bar{x} = \frac{\sum fx}{\sum f}$

$$= \frac{24941}{175} = 142.52$$

Standard Deviation $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

$$= \sqrt{\frac{3583693}{175} - \frac{24941}{175}}$$

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$$= \sqrt{20478.24571 - 20311.9504}$$

$= \sqrt{166.29531}$

S.D. $= 12.869$

For median and quartiles, we consider the following table:

Weight	C.B.	F	C.f	
118-126	117.5-126.5	20	20	
127-135	126.5-135.5	35	55	➔ Q_1
136-144	135.5-144.5	49	104	➔ Median
145-153	144.5-153.5	32	136	➔ Q_3
154-162	153.5-162.5	25	161	
163-171	162.5-171.5	14	175	

Median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$\frac{n}{2}$ $= \frac{175}{2} = 87.5$

l $= 135.5$

h $= 9$

f $= 49$

C $= 55$

Median $= 135.5 + \frac{9}{49}(87.5 - 55)$

$= 134.5 + 5.969$

Median $= 141.47$

Q_1 $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$\frac{n}{4}$ $= \frac{175}{4} = 43.75$

l $= 126.5$

h $= 9$

f $= 35$

C $= 20$

Q_1 $= 126.5 + \frac{9}{35}(43.75 - 20)$

$= 126.5 + 6.1071$

$Q_1 = 132.61$

$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$

$\frac{3n}{4} = \frac{3x175}{4} = 131.25$

$Q_3 = 144.5 + \frac{9}{32}(131.25 - 104)$

$= 144.5 + 7.664$

$Q_3 = 152.16$

QUESTION NO. 2

Commodity	2008		2012		p_nq_o	p_oq_o	p_nq_n	p_oq_n
	p_o	q_o	p_n	q_n				
A	5.0	80	8.7	100	696	400	870	500
B	3.6	90	5.7	95	513	324	541.5	342
C	3.1	20	4.6	30	92	92	138	93
					1301	**786**	**1549.5**	**935**

Laspreyr's price index $= \frac{\sum p_nq_o}{\sum p_oq_o}$ x 100

$= \frac{1301}{786}$ x 100

= 165,52

Paasche's price index $= \frac{\sum p_nq_n}{\sum p_oq_n}$ x 100

$= \frac{1549.5}{935}$ x 100

= 165.72

Marshall's price index $= \frac{\sum p_nq_o + \sum p_nq_n}{\sum p_oq_o + \sum p_oq_n}$ x 100

$= \frac{1301+1549.5}{786+935}$ x 100

$= \frac{280.5}{1721}$ x 100

= 165.63

Fisher's price index $= \sqrt{Laspeyre's\ index \times Paasche's\ index}$

$= \sqrt{165.52 \times 165.72}$

$= 165.621$

QUESTION NO. 3

(i) H_o: Income and type of school are independent.

H_I: Income and type of school are dependent.

(ii) Level of significance $= \alpha = 5\% = 0.05$

(iii) Test statistic following x^2 distribution at 1 d.f. $x^2 = \sum\frac{(O-E)^2}{E}$

(iv) **Critical Region:**

$$x^2_{cal} \geq x^2_{tab}$$

$$x^2_{cal} \geq 3.84$$

Income	Private	Government	Total
High	$\frac{1000x656}{1600} = 410$	$\frac{1000x944}{1600} = 590$	1000
Low	$\frac{600x656}{1600} = 246$	$\frac{600x944}{1600} = 354$	600
	656	944	1600

Calculation of x^2 value is shown below

O (Observed Frequency)	E (Expected Frequency)	O – E	$(O-E)^2$	$(O-E)^2/E$
494	410	84	7056	17.20976
506	590	-84	7056	11.95932
162	246	-84	7056	28.68293
438	354	84	7056	19.93220
				77.78421

Conclusion: Since x^2_{cal}= 77.78 > 3.84 so it falls in critical region, we reject H_o and conclude that income and type of school are dependent.

QUESTION NO. 4

(i)

Population elements = 0, 3, 6, 12, 15,18

Population Size = N = 6

Sample Size = n= 3

Let x denotes the element of population

X	X^2
0	0
3	9
6	36
12	144
15	225
18	324
$\sum X=54$	$\sum X^2=738$

Population mean μ $= \frac{\sum X}{N} = \frac{54}{6} = 9$

Population Variance = $\sigma^2 = \frac{\sum X^2}{N} - \left(\frac{\sum X}{N}\right)^2$

$\sigma^2 = \frac{738}{6} - (9)^2 = 123\text{-}81 = 42$

Number of all possible samples without replacement = ${}^{N}C_{n} = {}^{6}C_{3} = 20$

Sample with corresponding means are given below:

Samples	Sample mean ($\bar{x}$)	Samples	Sample mean ($\bar{x}$)
0, 3, 6	3	3, 6, 12	7
0, 3, 12	5	3, 6, 15	8
0, 3, 15	6	3, 6, 18	9
0, 3, 18	7	3, 12, 15	10
0, 6, 12	6	3, 12, 18	11
0, 6, 15	7	3, 15, 18	12
0, 6, 18	8	6, 12, 15	11
0, 12, 15	9	6, 12, 18	12
0, 12, 18	10	6, 15, 18	13
0, 15, 18	11	12, 15, 18	15

Sampling distribution of means is given below:

$\bar{x}$	f	$P(\bar{x})$	$\bar{x}.P(\bar{x})$	$\bar{x}^2.P(\bar{x})$
3	1	1/20	3/20	9/20
5	1	1/20	5/20	25/20
6	2	2/20	12/20	72/20
7	3	3/20	21/20	147/20
8	2	2/20	16/20	128/20
9	2	2/20	18/20	162/20
10	2	2/20	20/20	200/20
11	3	3/20	33/20	363/20
12	2	2/20	24/20	288/20
13	1	1/20	13/20	169/20
15	1	1/20	15/20	225/20
	$\sum f = 20$		**180/20**	**1788/20**

Mean of sampling distribution of means

$$\mu_{\bar{x}} = \bar{x}.P(\bar{x}) = \frac{180}{20} = 9$$

Variance of sampling Distribution

$$\sigma_{\bar{x}}^2 = \sum \bar{x}^2 P(\bar{x}) - [\sum \bar{x} P(\bar{x})]^2$$

$$= \frac{1788}{20} - (9)^2$$

$$= 89.4 - 81$$

$$\sigma_{\bar{x}}^2 = 8.4$$

(i) Relationship between sampling distribution of the means $\mu_{\bar{x}}$ and population mean (μ) is:

Verification:

$$\mu_{\bar{x}} = \mu$$

Verification:

$$\mu_{\bar{x}} = 9, \mu = 9$$

Hence, $\mu_{\bar{x}} = \mu$ (Both are equal)

(ii) Relationship between variance of sampling distribution ($\sigma_{\bar{x}}^2$) and population variance σ^2 is:

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$$

Verification: Since, $\sigma_{\bar{x}}^2 = 8.4$

And $\frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

$= \frac{42}{3} \times \frac{6-3}{6-1}$

$= 14 \times \frac{3}{5}$

$= 8.4$

So,

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1} = 8.4$ **Verified**

QUESTION NO. 5

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 4 & -2 & 7 \\ 3 & 2 & -4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 5 \\ 4 & -2 & 7 \\ 3 & 2 & -4 \end{vmatrix}$$

Expanding from 1st Row:

$$|A| = 1\begin{vmatrix} -2 & 7 \\ 2 & -4 \end{vmatrix} - 3\begin{vmatrix} 4 & 7 \\ 3 & -4 \end{vmatrix} + 5\begin{vmatrix} 4 & -2 \\ 3 & 2 \end{vmatrix}$$

= 1 (8-14) – 3 (-16-21) + 5(8+6)

= 1 (-6) – 3 (-37) + 5(14)

= -6+111+70

$|A| = 175$

$$\text{Adj A} = \begin{pmatrix} \begin{vmatrix} -2 & 7 \\ 2 & -4 \end{vmatrix} & -\begin{vmatrix} 4 & 7 \\ 3 & -4 \end{vmatrix} & +\begin{vmatrix} 4 & -2 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 3 & 5 \\ 2 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 3 & -4 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 5 \\ -2 & 7 \end{vmatrix} & -\begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 4 & -2 \end{vmatrix} \end{pmatrix}^t$$

$$= \begin{pmatrix} (8-14) & -(-16-21) & (8+6) \\ -(-12-10) & (-4-15) & -(2-9) \\ (21+10) & -(7-20) & (-2-12) \end{pmatrix}^t$$

$$= \begin{bmatrix} -6 & 37 & 14 \\ 22 & -19 & 7 \\ 31 & 13 & -14 \end{bmatrix}^t$$

After taking Transpose

$$\text{Adj A} = \begin{bmatrix} -6 & 22 & 31 \\ 37 & -19 & 13 \\ 14 & 7 & -14 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj. (A)}$$

$$= \frac{1}{175} \begin{bmatrix} -6 & 22 & 31 \\ 37 & -19 & 13 \\ 14 & 7 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-6}{175} & \frac{22}{175} & \frac{31}{175} \\ \frac{37}{175} & \frac{-19}{175} & \frac{13}{175} \\ \frac{14}{175} & \frac{7}{175} & \frac{-14}{175} \end{bmatrix}$$

QUESTION NO. 6

(a) $4x - 3y = 10$

$5x - 7y = 6$

Multiplying eq. (1) by 5 and eq. (2) by 4 and then subtracting the resultant equations:

Or (i) $20x - 15y = 50$

(ii) $\underline{-20x \pm 28y = -24}$

$13y = 26$

$y = \frac{26}{13} = 2$

By putting the value of y in equation (i):

$4x - 3(2) = 10$

$4x - 6 = 10$

$4x = 10 + 6$

$4x = 16$

$x = 16/4$

$x = 4$

Hence, Solution set = {[4, 2]}

QUESTION NO. 6

(b) Let,

Length of rectangular plot = x yards

Width of rectangular plot = y yards

Area of rectangular plot = 2000 sq.yards

$xy = 2000$ ………….(1)

Total length of the fencing = 180 yards = Perimeter

$2x+2y = 180$

$2(x+y) = 180$

$X+Y = 180/2$

$X+y = 90$

From eq. (2)

$Y = 90-x$

Putting in rq. (1):

$X(90-x) = 2000$

$90x - x^2 = 2000$

Or $x2 - 90x + 2000 = 0$

$a = 1$

$b = -90$

$c = 2000$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-90) \pm \sqrt{(-90)^2 - 4(1)(2000)}}{2(1)}$$

$$= \frac{90 \pm \sqrt{8100 - 8000}}{2}$$

$$= \frac{90 \pm \sqrt{100}}{2}$$

$$= \frac{90 \pm 10}{2}$$

$$X = \frac{90+10}{2}, \quad X = \frac{90-10}{2}$$

$$= \frac{100}{2}, \quad = \frac{80}{2}$$

50 40

For X = 50

Putting in y = 90-x

= 90 – 50

Y =40

For X = 40

Putting in y = 90-x

= 90 – 40

Y = 50

Hence,

If length of the plot = 50 yards

Then width of the plot = 40 Yards

And if

If length of the plot = 40 yards

Then width of the plot = 50 Yards

QUESTION NO. 7

(a) The given geometric series is:

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \ldots\ldots\ldots.$$

a = 1

Common ratio = $\frac{-1/2}{1} = -\frac{1}{2} = r$

Since, $|r| = |-1/2| < 1$

So, we use the formula for sum as:

$$S_n = a\left(\frac{a-rn}{1-r}\right)$$

Where a = 1

r = $-\frac{1}{2}$

n = 10

$$S_{10} = \frac{1-(-\frac{1}{2})10}{1-(-\frac{1}{2})}$$

$$= \left(\frac{1-\frac{1}{2}10}{1+\frac{1}{2}}\right)$$

$$= \frac{(1 - \frac{1}{1024})}{\frac{3}{2}}$$

$$= \frac{(\frac{1024-1}{1024})}{\frac{3}{2}}$$

$$= \left(\frac{1023}{1024}\right) \times \frac{3}{2}$$

$$S_{10} = \left(\frac{341}{512}\right)$$

Hence, Sum of 10 terms of geometric series is $\frac{341}{512}$

(b) 1st alternative of the executive = Rs. 240,000

2nd Alternative can be written in monthly sequence for first month , 2nd month , 3rd month, etc as below:

Rs. 100, Rs. 200, Rs. 400,…… upto 12 terms.

This makes a geometric sequence where:

$a = 100$

$r = \frac{200}{100} = 2$

$n = 12$

Sum of 12 month salary = $S_n = \frac{a(r^n - 1)}{r-1}$

$$= \frac{100(2^{12} - 1)}{2-1}$$

$= 100\ (4096-1)$

$= 100(4095)$

$=$ Rs. 409,500

Per year salary for 1st alternative = Rs. 240,000

Per year salary for 2nd alternative = Rs. 409,500

Since, Rs. 409,500 > Rs. 240,000

The executive should prefer 2nd alternative.

QUESTION NO. 8

(a) Principal amount = P = Rs. 4,500

Rate of interest for 1st year r = 4%

Compound amount after first year $= P(1+r)^n$

Where m = 1 We have: $= 4500\,(1+0.04)^1$

$= 4500\,(1.04)$

= Rs. 4680

Principal amount for 2nd Year = P = Rs. 4680

Rate of interest for 2nd year r = 5%

Compound amount after 2nd year $= P(1+r)^n$

$= 4680\,(1+0.05)$

$= 4680\,(1.05)$

= Rs. 4914

Principal amount for 3rd Year = P = Rs. 4914

Rate of interest for 3rd year r = 6%

Compound amount after 3rd year $= P(1+r)^n$

$= 4914\,(1+0.06)$

$= 4914(1.06)$

= Rs. 5208.84

Hence, Compound interest for 3 Years = 5208.84 – 4500

= Rs. 708.84

(b) Since the investment of equal size made at the end of each quarter, and we have to find the accumulated value. The problem is related to sum of ordinary annuity we have

R = Rs. 5000 (Each Periodic Payment)

I = 8%/4 = 2% = 0.02 (Quarterly Interest Rate)

n- 5×4 = 20 Quarters (Number of conversion periods)

So, the required accumulated value is given by the formula:

$S_n = R[\frac{(1+i)^n - 1}{i}]$

$S_n = 5000[\frac{(1+0.02)^{20} - 1}{0.02}]$

$S_n = 5000[\frac{(1.02)^{20} - 1}{0.02}]$

S_n = Rs. 121486.849