Solved Paper 2016

QUESTION NO. 1

Classes	f	x	fx	fx^2	Cumulative Frequency
0-50	3	25	75	1875	3
50-100	7	75	525	39375	10
100-150	12	125	1500	187500	22
150-200	18	175	3150	551250	40
200-250	21	225	4725	1063125	61
250-300	12	275	3300	907500	73
	73		13275	2750625	

Median $= l + \frac{i}{f}(\frac{\sum f}{2} - c)$

$= 150 + \frac{50}{18}(36.5 - 22) = 190.28$

Mode $= l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} x\ i$

$= 200 + \frac{(21-18)}{(21-18) + (21-12)} x\ 50$

$= 200 + \frac{3x50}{3+9} = 200 + 12.5 = 212.25$

$Q_1 = l + \frac{i}{f}(\frac{n}{4} - c)$

$= 100 + \frac{50}{12}(18.25 - 10)$

$= 100 + \frac{50}{12}(8.25)$

$= 100 + 34.375 \quad = 134.375$

$Q_3 = l + \frac{i}{f}(\frac{3n}{4} - c)$

$= 200 + \frac{50}{21}(54.75 - 40)$

$= 200 + \frac{50}{21}(14.75)$

$= 200 + 35.119 \quad = 235.119$

$$S.K = \frac{Q_3+Q_1-2Median}{Q_3+Q_1}$$

$$S.K = \frac{235.119+134.375-2x190.278}{235.119-134.375}$$

$$S.K = \frac{369.494-380.556}{100.744} = \frac{-11.062}{100.744} = -0.11$$

(or)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{13275}{73} = 181.849$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{2750625}{73} - \left(\frac{13275}{73}\right)^2}$$

$$= \sqrt{37679.79452 - 33069.17339}$$

$$= \sqrt{4610.62113} = 67.9$$

$$S.K = \frac{Mean-mode}{\sigma}$$

$$= \frac{181.849-212.25}{67.9} = \frac{-30.651}{67.9} = -0.45$$

(OR)

$$S.K = \frac{3(Mean-Median)}{\sigma}$$

$$= \frac{3(181.849-190.278)}{67.9} = -0.372$$

QUESTION NO. 2

Items	2005		2012		p_nq_o	p_oq_o	p_nq_n	p_oq_n
	p_o	q_o	p_n	q_n				
A	17	135	27.52	369	3715.2	2295	10154.88	6273
B	19.36	214	29.59	247	6332.26	4143.04	7308.73	4781.9
C	15.18	191	14.46	227	2761.86	2899.38	3282.42	3445.86
D	99.32	161	96.17	186	15483.37	15990.52	17887.62	18473.52
					28292.69	25327.94	38633.65	32974.3

Index Number Using:

(i) Laspreyr's index $= \frac{\sum p_n q_o}{\sum p_o q_o}$ x 100

$= \frac{28292.69}{25327.96}$ x 100 = 111.71

Paasche's index $= \frac{\sum p_n q_n}{\sum p_o q_n}$ x 100

$= \frac{38633.65}{32974.30}$ x 100 = 117.16

Fisher's index $= \sqrt{\frac{\sum p_n q_o}{\sum p_o q_o} \times \frac{\sum p_n q_n}{\sum p_o q_n}}$ x 100

$= \sqrt{\frac{28292.69}{25327.96} \times \frac{38633.65}{32974.30}}$ x 100 = 114.4

QUESTION NO. 3

X	Y	X^2	Y^2	XY
16	40	256	1600	640
72	51	5184	2704	3744
73	43	5329	1849	3139
63	49	3969	2401	3087
83	61	6889	3721	5063
80	58	6400	3364	4640
66	44	4356	1936	290
66	58	4356	3364	3828
74	50	5476	2500	3700
62	45	3844	2025	2790
$\sum X = 656$	$\sum Y = 500$	$\sum X^2 = 46059$	$\sum Y^2 = 25464$	$\sum XY = 33535$

$\sigma_x = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$

$= \sqrt{\frac{46059}{10} - \left(\frac{655}{10}\right)^2}$

$= \sqrt{4605.9 - 4290.25}$

$= \sqrt{315.65} = 17.77$

$\sigma_y = \sqrt{\frac{\sum Y^2}{n} - \left(\frac{\sum Y}{n}\right)^2}$

$= \sqrt{\frac{25464}{10} - \left(\frac{500}{10}\right)^2}$

$= \sqrt{2546.4 - 2500}$

$= \sqrt{46.4} = 6.81$

$r = \frac{\frac{\sum xy}{n} - \frac{\sum x}{n} \times \frac{\sum y}{n}}{\sigma_x \sigma_y}$

$= \frac{\frac{33535}{10} - \frac{655}{10} \times \frac{5500}{10}}{17.77 \times 681}$

$= \frac{3353.5 - 3275}{121.0137}$

$= \frac{78.5}{121.0137} = 0.649 \text{ or } 0.65$

Line if regression y on x:

$y - \bar{y} = r\frac{\sigma_x}{\sigma_y}(x - \bar{x})$

$y - 50 = 0.65 \text{ x } \frac{6.81}{17.77}(x - 65.5)$

$y = 50 - 16.31 + 0.249x$

$y = 33.69 + 0.249x$

QUESTION NO. 4

(i)

Population elements = 8, 12, 16, 8, 20

X	X^2
8	64
12	144

16	256
18	324
20	400
74	1188

Population mean $\mu = \frac{\sum X}{N} = \frac{74}{5} = 14.8$

Standard deviation of population = $\sigma = \sqrt{\frac{\sum X^2}{N} - \left(\frac{\sum X}{N}\right)^2}$

$\sigma = \sqrt{\frac{1188}{5} - \left(\frac{74}{5}\right)^2} = \sqrt{237.6 - 219.04} = \sqrt{18.56}$

$= 4.308131846 \text{ or } 4.308$

(ii)

Samples	Sample mean ($\bar{x}$)	$\bar{x}$	Tally	f	$f\bar{x}$	$f(\bar{x})^2$
8, 12	10	10	1	1	10	100
8, 16	12	12	1	1	12	144
8, 18	13	13	1	1	13	169
8, 20	14	14	11	2	28	392
12, 16	14	15	1	1	15	225
12, 18	15	16	1	1	16	256
12, 20	16	17	1	1	17	289
16, 18	17	18	1	1	18	324
16, 20	18	19	1	1	19	361
18, 20	19					
				10	148	2260

$\mu_{\bar{x}} = \frac{\sum f\bar{x}}{\sum f} = \frac{148}{10} = 14.8$

$\sigma_{\bar{x}}^2 = \sqrt{\frac{\sum f(\bar{x})^2}{\sum f} - \left(\frac{\sum f\bar{x}}{\sum f}\right)^2}$

$\sigma = \sqrt{\frac{2260}{10} - \left(\frac{148}{10}\right)^2}$

$= \sqrt{226 - 219.04} = \sqrt{6.96}$

$= 2.638181192 \text{ or } 2.638$

Verification:

$\mu_{\bar{x}} = \mu$

$14.8 = 14.8$

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} . \frac{N-n}{N-1}$

$(2.638)^2 = \frac{(4.308)^2}{2} \times \frac{5-2}{5-1}$

$6.96 = \frac{(4.308)^2}{2} \times \frac{3}{4}$

$6.96 = 6.96$

SECTION-II

QUESTION NO. 5

(a) The given infinite geometric series:

$$5 + \frac{5}{6} + \frac{5}{36} + \ldots.. \alpha$$

Here, $a = 5$, $r = \frac{5}{6} \div 5 = \frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$

$S_\alpha = \frac{a}{1-r} = \frac{5}{1-\frac{1}{6}} = \frac{5}{\frac{5}{6}} = 5 \text{ x } \frac{6}{5} = 6$

(b) Which the, of the sequence 16, 8, 4, 2, …… is $\frac{1}{16}$

$a = 16$, $r = \frac{8}{16} = \frac{1}{2}$

let $\frac{1}{16}$ is nth term:

$ar^{n-1} = \frac{1}{16}$

$16\left(\frac{1}{16}\right)^{n-1} = \frac{1}{16x16} = \frac{1}{256} = \frac{1}{(2)^8} = \left(\frac{1}{2}\right)^8$

$n - 1 = 8$

$n = 8 + 1 = 9$ $\left(So\ \frac{1}{16}\ is\ 9th\ term\right)$

QUESTION NO. 6

(a) $\sqrt{5x+4} - \sqrt{3x+1} = 1$

$\sqrt{5x+4} = (1 + \sqrt{3x+1})^2$

Taking square on both sides:

$(5x+4) = 1 + 3x + 1 + 2 \times 1\sqrt{3x+1}$

$5x + 4 - 1 - 3x - 1 = 2\sqrt{3x+1}$

$2x + 2 = 2\sqrt{3x+1}$

$x + 1 = \sqrt{3x+1}$

Taking the square on both sides:

$x^2 + 2x + 1 = 3x + 1$

$x^2 + 2x - 3x + 1 - 1 = 0$

$x^2 - x = 0$

$x(x - 1) = 0$

So, $x = 0 \Rightarrow x = 0$

or $x - 1 = 0 \Rightarrow x = 1$

Solution set is $\{0, 1\}$

(b) $5x + 4y = 7$

$3x - 4y = 17$

$8x = 24$

$x = 3$

$5x + 4y = 7$

$5 \times 3 + 4y = 7$

$15 + 4y = 7$

$4y = 7 - 15 = -8$

$y = \frac{-8}{4} = -2$

Solution set of {x = 3 and y = –2}

QUESTION NO. 7

$A^{-1} = \left(\frac{1}{|A|}\right)$ adjoint A

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

Co-factors of A are:

$A_{11} = (-1)^2\begin{vmatrix} -1 & 2 \\ 3 & 1 \end{vmatrix} = 1(1-6) = -5$

$A_{12} = (-1)^3\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1(-1-4) = 5$

$A_{13} = (-1)^4\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 1(3-2) = 5$

$A_{21} = (-1)^3\begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1(-1-3) = 4$

$A_{22} = (-1)^4\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = 1(-1-2) = -3$

$A_{23} = (-1)^5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1(3-2) = -1$

$A_{31} = (-1)^4\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 1(2+1) = 3$

$A_{32} = (-1)^5\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1(2-1) = -1$

$A_{33} = (-1)^6\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1-1) = -2$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= (1 \text{ x } (-5) + 1 \text{ x } 5 + 1 \text{ x } 5)$

$= -5 + 5 + 5 = 5 \neq 0$

$$A_C = \begin{bmatrix} -5 & 5 & 5 \\ 4 & -3 & -1 \\ 3 & -1 & -2 \end{bmatrix} \text{ and Adj A} = \begin{bmatrix} -5 & 4 & 3 \\ 5 & -3 & -1 \\ 5 & -1 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} -5 & 4 & 3 \\ 5 & -3 & -1 \\ 5 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4/5 & 3/5 \\ 1 & -3/5 & -1/5 \\ 1 & -1/5 & -2/5 \end{bmatrix}$$

QUESTION NO. 8

We have

$P = \text{Rs. } 800{,}000$

$i = \frac{16}{100} \times \frac{1}{2} = 0.08$

$n = 10 \text{ x } 2 = 20 \text{ (Payments)}$

$R = ?$

$$R = P\left[\frac{i}{1-(1-i)^{-n}}\right]$$

$$= 800{,}000\left[\frac{0.08}{1-(1-0.08)^{-20}}\right]$$

$$= \frac{64{,}000}{1-0.214548207} = \frac{64{,}000}{0.785451792}$$

$= 81481.76706 \text{ or } 81481.77$